Detecting d-wave superfluid and d-density wave states of ultracold Fermions on optical lattices

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We propose a pump probe experiment for detecting the d-wave superfluid and d-density wave phases of ultracold Fermions on an optical lattice. The pump consists of periodic modulations of the optical lattice intensity which creates quasiparticle pairs in these systems. The changes in the momentum distribution under the drive can be used to measure quasiparticle dispersion and gap anisotropy. Further, we show that the pattern of peaks and dips in the spin selective density-density correlation function provides a phase sensitive probe of the symmetry of the order parameter in these systems.

Ultracold atoms on optical lattices are emerging as candidates for building a *quantum simulator* [1] of various lattice models used to study strongly interacting many body systems. These systems have already been used to implement important lattice Hamiltonians like the Bose Hubbard model [2], which is central to the theory of superfluid-insulator transitions [3] and attractive and repulsive Fermi Hubbard models, which are central to the study of strongly interacting superfluids [4] and high temperature superconductors [11] respectively. The recent observation of Mott insulating states in the repulsive Fermi Hubbard model [5, 6] shows a novel manifestation of strong interaction in these systems.

The attractive feature of cold atom systems is that they provide a clean setting with controlled and tunable access to the strongly interacting parameter regime of model Hamiltonians. However, a good quantum simulator also requires methods to extract information about the many body correlations in the system. This is specially relevant for simulating strongly correlated systems with complex phase diagrams, high T_c superconductors, where the quantum simulation can shed light on the nature of the various phases and test the various competing theories that have been proposed. A quantum simulator which aims to choose between competing theoretical descriptions should be able to probe the characteristic correlations and excitations of the various observed and proposed phases.

In the setting of strongly correlated system condensed matter setting like high T_c superconductors, measurement of thermodynamic quantities like specific heat and of transport properties like resistivity, Hall conductivity, and Nerst effect as well as various spectroscopic techniques like neutron scattering and optical conductivity have been used to identify the various thermodynamic phases and have yielded a lot of information about correlations and excitations. Detailed information about the electronic structure obtained from ARPES [14] and STM [15, 16] measurements has been crucial in identifying the nature of these thermodynamic phases. A crucial aspect of constructing the phase diagram is to identify the non-trivial symmetry of the various order parameters e.g. the d-wave symmetry of the superconductor. The techniques mentioned above contain indirect information about the symmetry, but a more reliable method for probing the symmetry of the superconducting order parameter has been to perform a phase-sensitive measurement, i.e. look for a π phase-shift in the corner junction SQUID [13]. However, this method is not directly applicable to the cold atom setting. Hence, it is important to develop new experimental techniques for cold atom systems to probe the correlations and order parameter symmetry.

In this Letter, we consider the problem of characterizing d-wave Fermionic superfluids and states with d-density wave order. The d-density wave state has been proposed as a possible candidate for the pseudogap phase [9]. We focus on two issues: (1) determining the quasi-particle dispersion and identifying the location of the nodes in the gap function if there are any and (2) experimentally identifying the order parameter symmetry. Our method consists of looking at the response of the system to periodic modulation of the optical lattice intensity. This drive creates pairs of quasi-particles of opposite momenta predominantly with total energy corresponding to the frequency of the drive. The change in momentum distribution n_k in response to the drive can be used to measure the quasiparticle dispersion. Further, the pattern of peaks and dips in different density-density correlation functions, which result from the interference of the quasiparticles excited by the drive, encode the symmetry of the order parameter (e.g. s-wave vs. d-wave pairing). The momentum distribution can be measured by a ballistic expansion experiment, while the correlators can be measured by noise correlations in the real space density profiles. The Letter consists of two parts: in the first part, we describe our method and apply it to the d-wave superfluid case, and in the second part we apply it to the d-density wave case.

d-wave Superfluid — We will use the simplified BCS description of a d-wave superfluid, which helps in demonstration of our proposed methods; however the qualitative results that we obtain are independent of the microscopic description, and the non-BCS nature of the superfluid will only result in quantitative discrepancies. Thus this method may be applied to the superfluid phase of the Fermi-Hubbard model (assuming that it exists). We start with the tight binding BCS Hamiltonian on a two dimensional square lattice, suitably generalized for arbitrary pairing symmetry,

$$H_0 = \sum_{\mathbf{k}} \begin{pmatrix} c^\dagger_{\mathbf{k}\uparrow} & c_{-\mathbf{k}\downarrow} \end{pmatrix} \begin{pmatrix} \xi_{\mathbf{k}} & \Delta_{\mathbf{k}} \\ \Delta_{\mathbf{k}} & -\xi_{\mathbf{k}} \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}\uparrow} \\ c^\dagger_{-\mathbf{k}\downarrow} \end{pmatrix} \qquad (1)$$

where $c^\dagger_{\mathbf{k}\sigma}$ is the creation operator for an atom with flavor $\sigma \in \{\uparrow, \downarrow\}$ and momentum $\mathbf{k}$ and $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ is the tight binding dispersion with μ the chemical potential and

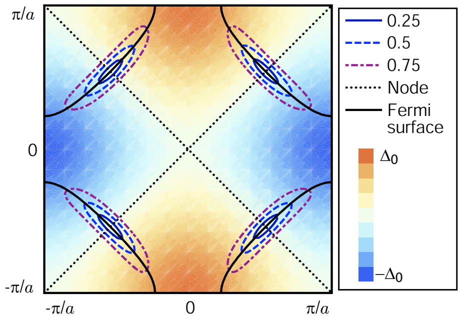

FIG. 1: Brillouin zone of a tight binding d-wave BCS superfluid. Solid black lines indicate the location of the Fermi surface (without the pairing interaction), while the dotted black lines indicate the location of the nodes. The shading indicates the value of order parameter $\Delta_{\mathbf{k}}$. Several equal quasiparticle energy contours, *bananas*, are shown with colored lines.

$\epsilon_{\mathbf{k}} = -2J(\cos k_x + \cos k_y)$ where J is the tunneling matrix element. The mean field $\Delta_{\mathbf{k}}$ describes the pairing interaction, and its $\mathbf{k}$ dependence reflect the pairing symmetry. We focus on the case of d-wave pairing $\Delta_{\mathbf{k}} = \Delta_0(\cos k_x - \cos k_y)$.

We drive the system by periodic modulation of the optical lattice intensity $V(t) = V_0 + \delta V \sin(\omega t)$, which modulates the tunneling matrix element $J \to J(1+g_1(t))$. The modulation of the superconducting gap $\Delta_0 \to \Delta_0(1+g_2(t))$ depends on the microscopic mechanism of pairing. However, the precise form of $g_2(t)$ is not important for qualitative results; the important point is that both $g_1(t)$ and $g_2(t)$ have similar time dependence ($\sim \sin(\omega t)$) [17]. (For the purposes of the example calculations we set $g_2(t) = 2g_1(t) = 2\lambda \sin(\omega t)$, where $\lambda \sim \sqrt{V_0/E_r}\delta V_0/V$. This choice is motivated by the expectation that $\Delta \sim J^2/U$ [11].)

For illustrative purposes, we shall consider driving a system that starts at zero temperature, i.e. with the initial condition that all quasiparticle states are empty. As the perturbation Hamiltonian only creates or destroys quasiparticles in pairs, we truncate the Hilbert space to exclude states with unpaired quasiparticles. This truncated Hilbert space is spanned by wavefunctions of the BCS form

$$\Psi(t) = \prod_{\mathbf{k}} \left(u_{\mathbf{k}}(t) + v_{\mathbf{k}}(t) c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow} \right) |0\rangle. \qquad (2)$$

The initial state corresponds to the BCS solution $u_{\mathbf{k}} = \mathrm{sign}(\Delta_{\mathbf{k}}) \left[\frac{1}{2}(1+\xi_{\mathbf{k}}/E_{\mathbf{k}})\right]^{1/2}$ and $v_{\mathbf{k}} = \left[\frac{1}{2}(1-\xi_{\mathbf{k}}/E_{\mathbf{k}})\right]^{1/2}$ with $E_{\mathbf{k}} = \sqrt{\xi^2_{\mathbf{k}} + \Delta^2_{\mathbf{k}}}$ the quasiparticle dispersion of the unperturbed state. The equations of motion for the coherence factors are

$$i\partial_t \begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix} = \begin{pmatrix} 0 & \Delta_{\mathbf{k}}(t) \\ \Delta_{\mathbf{k}}(t) & 2(\xi_{\mathbf{k}} + g_1(t)\epsilon_{\mathbf{k}}) \end{pmatrix} \begin{pmatrix} u_{\mathbf{k}} \\ v_{\mathbf{k}} \end{pmatrix}. \qquad (3)$$

where $\Delta_{\mathbf{k}}(t) = \Delta_{\mathbf{k}}(1+g_2(t))$. In describing the evolution, we do not maintain the self-consistency equations for Δ and μ. This is justified in the linear response regime where we do not create a lot of quasiparticles.

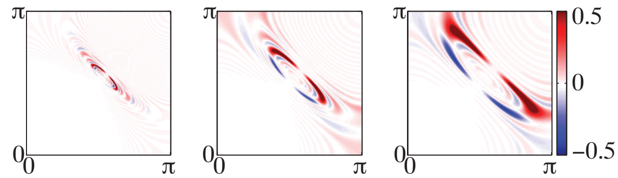

FIG. 2: Change in the momentum distribution function under the influence of the drive $\langle n_k(t = 25/J)\rangle - \langle n_k(0)\rangle$ for a BCS d-wave superfluid, with $\Delta_0 = 0.5J$, for various drive frequencies: $\omega = \Delta_0/2$ (left), Δ_0 (center), $1.5\Delta_0$ (right).

To gain a better understanding of the dynamics, we use second order perturbation theory in λ to study the evolution of the system. It is more convenient to perform perturbation theory calculations in the quasiparticle basis

$$\Psi(t) = \prod_{\mathbf{k}} \left(\psi_{1,\mathbf{k}}(t) + \psi_{2,\mathbf{k}}(t) \gamma^\dagger_{\mathbf{k}\uparrow} \gamma^\dagger_{-\mathbf{k}\downarrow} \right) |\Psi_{\mathrm{BCS}}\rangle, \qquad (4)$$

where $\gamma^\dagger_{\mathbf{k},\sigma} = u_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} + \sigma v_{\mathbf{k}} c_{-\mathbf{k},\bar{\sigma}}$ are the quasiparticle creation operators. In the tight binding d-wave superfluid, the contours of equal quasiparticle energy look like *bananas* (See Fig 1). When $\mathbf{k}$ is off resonance (i.e. away from the banana contour corresponding to $\omega = 2E_{\mathbf{k}}$), with detuning $\tilde{\omega}_{\mathbf{k}} = 2E_{\mathbf{k}} - \omega$, $|\psi_{2,\mathbf{k}}|$ oscillate between 0 and $\sim \lambda u_{\mathbf{k}} v_{\mathbf{k}} \epsilon_{\mathbf{k}}/\tilde{\omega}_{\mathbf{k}}$ at approximately the drive frequency. On the other hand, if $\mathbf{k}$ is on resonance, then $|\psi_{2,\mathbf{k}}|$ oscillate between 0 and 1 with the period $P_{\mathbf{k}} \sim (2\lambda u_{\mathbf{k}} v_{\mathbf{k}} \epsilon_{\mathbf{k}})^{-1}$. Thus, after the system has been driven for some time, the quasi-particle momentum distribution function $\langle \gamma^\dagger_{\mathbf{k}} \gamma_{\mathbf{k}} \rangle$ will develop peaks along the banana contours corresponding to $\omega = 2E_{\mathbf{k}}$, with maxima near the banana tips. To optimize the peak height, the system should be driven for a time $t \sim P^{-1}_{\mathbf{k}}$ for $\mathbf{k}$ near a banana tip. For the remainder of the paper, we shall solve the equations of motion numerically, but use the insights from this simple perturbation theory to qualitatively explain the results.

Momentum distribution function n_k — To extract information about the dispersion relation, we propose to look at the momentum distribution of the atoms, $n_k = \langle c^\dagger_{\mathbf{k}} c_{\mathbf{k}} \rangle = 2|v_{\mathbf{k}}(t)|^2$. Experimentally, n_k may be obtained from a band-mapping free expansion experiment [18]. As an illustration, we consider the case of a d-wave superfluid ($\mu = 4.5$, $\Delta_0 = 1.0$, $J = 1$) that we drive at three different frequencies $\omega = \{0.5, 1.0, 1.5\}\Delta_0$. The change in momentum distribution at a time $t = 25$ is plotted in Fig 2. As explained before, at a given frequency ω, the excitations are being driven predominantly on the banana-shaped contour corresponding to $\omega = 2E_{\mathbf{k}}$. This leads to large changes in the momentum distribution along the same contour. The dispersion can then be extracted by varying the drive frequency. If the frequency decreases (increases), the banana-shaped contour shrinks (grows) as can be seen in the left (right) panel of Fig. 2. The fringes correspond to the diffraction pattern that is formed at short times. The dispersion will reveal the $\mathbf{k}$ depen-

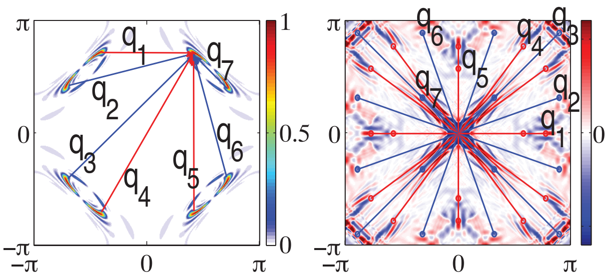

FIG. 3: Left: Quasiparticle density $|\psi_{\mathbf{k},2}(t = 25/J)|^2$. Red (blue) arrows indicate wave-vectors that connect tips of bananas between which $\Delta_{\mathbf{k}}$ does not change (changes) sign. Right: Correlation function $\langle\rho_\uparrow(q)\rho_\downarrow(-q)\rangle_t - \langle\rho_\uparrow(q)\rho_\downarrow(-q)\rangle_0$ for a d-wave superfluid. Pairing symmetry is encoded in peaks (red) and dips (blue) that develop in the correlation function as described in the text. (for both panels $\omega = \Delta_0$)

dent gap function and hence the d-wave nature of the superfluid. In this respect, this method is similar to the ARPES determination of the momentum dependence of the gap in high T_c superconductors [19].

In this context we will briefly comment on the response of a s-wave superfluid to the same drive. Since the s-wave system is gapped, one would not observe any response before the drive frequency exceeds twice the gap. In contrast, the low energy nodal quasiparticles in a d-wave superfluid lead to response at all drive frequencies.

Spin up–spin down density correlation function — Next, we show that the changes in the spin up–spin down density correlation function $N^{\uparrow\downarrow}(q) = \langle\rho_\uparrow(q)\rho_\downarrow(-q)\rangle$ show a pattern of peaks and dips which reveals the order parameter symmetry in a way similar to corner junction SQUID measurements in condensed matter systems. This correlation function can either be measured using elastic light scattering or extracted from the noise in real space measurement of the particle density. The point of measuring this particular correlation function is that it is sensitive to the symmetry of the pairing interactions in the system.

We can relate $N^{\uparrow\downarrow}(q)$ to the BCS wavefunction at time t defined by the coherence $u_{\mathbf{k}}(t)$ and $v_{\mathbf{k}}(t)$ as follows

$$N^{\uparrow\downarrow}(q) = \sum_{\mathbf{k},\mathbf{k}'} \left\langle c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow}(t) c_{\mathbf{k}\uparrow}(t) c^\dagger_{\mathbf{k}'-\mathbf{q}\downarrow}(t) c_{\mathbf{k}'\downarrow}(t)\right\rangle \quad (5)$$

$$= \sum_{\mathbf{k}} \left\langle c^\dagger_{\mathbf{k}+\mathbf{q}\uparrow}(t) c^\dagger_{-\mathbf{k}-\mathbf{q}\downarrow}(t)\right\rangle \langle c_{-\mathbf{k}\downarrow}(t) c_{\mathbf{k}\uparrow}(t)\rangle \quad (6)$$

$$= \sum_{\mathbf{k}} u_{\mathbf{k}}(t) v^*_{\mathbf{k}}(t) u^*_{\mathbf{k}+\mathbf{q}}(t) v_{\mathbf{k}+\mathbf{q}}(t). \quad (7)$$

Here, in the middle step, we have used the fact that for the BCS wavefunction only the anomalous pair correlations are non-zero. We note that the d-wave symmetry of $u_{\mathbf{k}} v^*_{\mathbf{k}}$ is preserved throughout the time evolution of the system. The form of $N^{\uparrow\downarrow}(q)$ looks like the product of Cooper pair wavefunctions at $\mathbf{k}$ and $\mathbf{k}+\mathbf{q}$, and therefore measures the interference of quasiparticles created at the respective wavevectors. Driving creates predominantly creates quasiparticles near the tips of the resonant banana contours. The interference of these quasiparticles result in peaks or dips in the noise correlation function whenever the wave-vector $\mathbf{q}$ that appears in $N^{\uparrow\downarrow}(q)$ connects a pair of banana tips located at $\mathbf{k}$ and $\mathbf{k}+\mathbf{q}$.

The change of $N^{\uparrow\downarrow}(q)$ after driving the system for ~ 4 cycles is depicted in the right panel of Fig. 3 for the case of the d-wave superfluid. To decipher the meaning of the various peaks and dips we compare the right panel of Fig. 3 to the left panel which depicts the corresponding change in the quasiparticle distribution function $\langle\gamma^\dagger_{\mathbf{k}}\gamma_{\mathbf{k}}\rangle$. Connecting the tips of the bananas (or the peaks in the quasiparticle density) with arrows in all possible ways we identify 7 unique wave-vectors, which are labeled q_1 through q_7 in Fig. 3. The vectors that connect the remaining pairs of tips may be obtained by reflections of $q_1..q_7$ across the nodal directions. The presence of a peak/dip at a given $\mathbf{q}$ vector depends on the time of drive; however the relative pattern of peaks and dips is governed by the symmetry of the order parameter. If the wave vector connecting two banana tips where $\Delta_{\mathbf{k}}$ and $\Delta_{\mathbf{k}+\mathbf{q}}$ has the same sign (say $\mathbf{q}_1$) has a peak in the signal, the wave-vectors connecting the tips where the order parameter has opposite signs (say $\mathbf{q}_2$) should have a dip and vice versa. Note that for s-wave pairing where order parameter does not change sign we would have all peaks or all dips depending on time of drive. For other singlet pairing symmetries the relative pattern would change depending on the location in the Brillouin zone where the order parameter changes sign.

Therefore, given the noise correlation function we can (1) determine the dispersion relation along the Fermi surface by measuring how the distances between the peaks and the dips varies with the drive frequency; (2) determine the pairing symmetry by figuring out the relative sign of $\Delta_{\mathbf{k}}$ at the various banana tips from the pattern of dips and peaks.

The important point to note is that our method asks a binary question of whether there is a peak or a dip much like corner junction SQUID experiment. In this sense this method is robust to quantitative changes in the signal due to strong correlation effects.

d-Density Wave — Next, we show that similar methods can be applied to the d-density wave states which have been proposed as prospective candidates for the pseudogap phase of high T_c superconductors [9]. In this state the operator $c^\dagger_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}}$ acquires an expectation value $\Delta_{\mathbf{k}}$. We concentrate on the commensurate case in which the ordering occurs at momentum $\mathbf{q} = (\pi, \pi)$. At the mean field level the system is described by the Hamiltonian

$$H_0 = \sum_{\mathbf{k}} \xi_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + \Delta_{\mathbf{k}} c^\dagger_{\mathbf{k}} c_{\mathbf{k}+\mathbf{q}} + \Delta^*_{\mathbf{k}} c_{\mathbf{k}} c^\dagger_{\mathbf{k}+\mathbf{q}}, \quad (8)$$

where the mean field has the same form as for the d-wave superfluid case $\Delta_{\mathbf{k}} = \Delta_0(\cos(k_x) - \cos(k_y))$. The commensurate ordering halves the Brillouin zone but fills it with two different (quasiparticle) bands. The two bands touch at

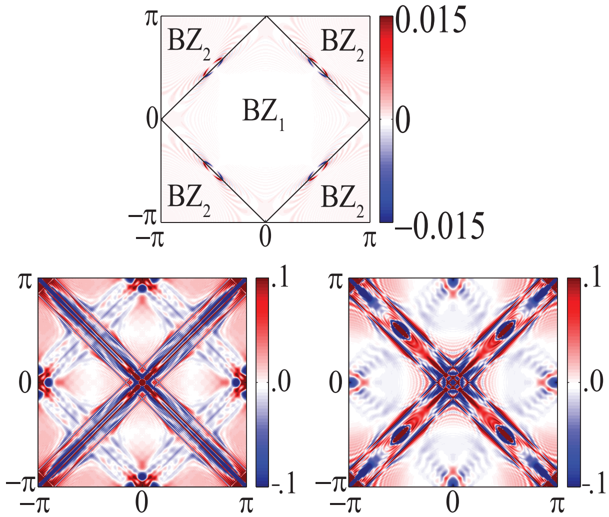

FIG. 4: Effects of drive at frequency $\omega = \Delta_0/2$ on density wave states. Change in the momentum distribution function $\langle n_{\mathbf{k}}(t = 25/J)\rangle - \langle n_{\mathbf{k}}(0)\rangle$ for d-density-wave at half-filling (top). Change in the density-density correlation function $\langle \rho_{\mathbf{k}}\rho_{\mathbf{q-k}}(t = 25/J)\rangle - \langle \rho_{\mathbf{k}}\rho_{\mathbf{q-k}}(t = 0)\rangle$ for anisotropic s-density-wave $\Delta_{\mathbf{k}} = 0.5|\cos(k_x) - \cos(k_y)|$ (left) and d-density-wave $\Delta_{\mathbf{k}} = 0.5\cos(k_x) - \cos(k_y)$ (right).

the points $(\pm\pi/2, \pm\pi/2)$. At half-filling ($\mu = 0$), the lower band is completely filled and the upper band is completely empty. Away from half filing there are empty states in the lower band. Optical lattice modulation excites quasiparticles from the lower branch to the upper branch leaving quasiholes behind in the lower band. In other words, in analogy with the d-wave superfluid, the drive creates quasihole-quasiparticle pairs with zero total momentum and total energy ω. The wave function in analogy to Eq. (2), can be written in the form

$$\Psi(t) = \prod_{\mathbf{k}} \left(\alpha_{\mathbf{k}}(t)c^\dagger_{\mathbf{k}} + \beta_{\mathbf{k}}(t)c^\dagger_{\mathbf{k+q}}\right). \tag{9}$$

The dispersion relation for quasiparticles can again be obtained by looking at the response of the momentum distribution to the drive. Because of the folding, the momentum distribution for the inner half-zone and outer half zone must be patched together

$$n_{\mathbf{k}} = \begin{cases} |\alpha_{\mathbf{k}}(t)|^2 & \mathbf{k} \in \text{inner zone} \\ |\beta_{\mathbf{k}}(t)|^2 & \mathbf{k} \in \text{outer zone} \end{cases} \tag{10}$$

There is one important difference in that tuning the system away from half-filling does not shift the location of the bananas as the density wavevector $\mathbf{q}$ has been assumed to be fixed. So independent of filling, the centers of the bananas remain fixed at $(\pm\pi/2, \pm\pi/2)$. Away from half-filling the low frequency response (inner portion of the banana) disappears as both branches are either empty or filled and thus a quasihole-quasiparticle pair cannot be created.

The symmetry of the order parameter is encoded in the pattern of peaks and dips of the $\langle \rho_{\mathbf{k}}\rho_{\mathbf{q-k}}\rangle$. We compare this pattern for two cases $\Delta_{\mathbf{k}} = \Delta_0(\cos(k_x) - \cos(k_y))$ and $\Delta_{\mathbf{k}} = \Delta_0|\cos(k_x) - \cos(k_y)|$. Because the peaks in the quasi-particle density near the tips of the bananas are not as sharp as for the case of d-wave superfluid it is hard to identify the origin of various peaks and dips, but there is a clear distinction between the two cases.

Concluding remarks — Probing the properties of the correlated state of ultracold atoms is an important part of building a "quantum simulator." In this Letter we propose to use periodic modulation of the optical lattice amplitude to probe Fermionic superfluids and density wave states. This probe is relevant to simulating the repulsive Hubbard model which is thought to be a minimal model of high T_c superconductors. The methods we suggest are useful for probing states with nodes or at least an anisotropic gap. We have shown that: First, we can determine the quasi-particle dispersion and gap anisotropy by looking at the change in the momentum distribution created by the drive. Second, by measuring the appropriate density-density correlation function we can determine the pairing symmetry from the pattern of peaks and dips that develops. By measuring the distances between the peaks and dips, we can also extract the gap anisotropy along the Fermi-surface from the second method method.

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